Crack size and strength distribution of structural ceramics after non-destructive inspection

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The crack-size d_c and the strength distributions of structural ceramics after non-destructive inspection (NDI) were analysed by supposing penny-shaped cracks to be inner cracks. The theoretical crack-size distribution function after NDI coincides well with the experimental data for a three-point bending test of HP-Si_3N_4 in the region $d_c > 30\,\mu m$. The distribution function of the fracture strength calculated from the crack-size data after NDI coincides well with the experimental data.

1. Introduction

It is well known that structural ceramics are extremely brittle and they catastrophically fail from the weakest defect within a body or lying on its surface such as a crack. Therefore in order to assure the structural reliability and to improve the strength of ceramics, it is essential to analyse the size distribution of the crack which may cause fracture of ceramics. In previous reports [1, 2], the authors suggested a new statistical theory of fracture location by combining the Oh–Finnie theory [3] with the competing risk theory, which enables us to estimate the crack-size distribution and the distribution of the fracture locations for multiple fracture origins.

On the other hand, the so-called "proof test" plays an important role in assuring the smallest strength (inert strength) or the smallest life-time (slow crack growth) [4, 5] of ceramic components; however, this method needs a lot of work and time.

Recently, screening techniques based on non-destructive inspection (NDI) using the ultrasonic microscope, Raman microscope or X-ray CT-scanner have been developing rapidly. However, quantitative analyses of the crack-size distribution and the strength distribution of the specimens after screening have not yet been performed.

In this report, we analyse the crack-size distribution and the strength distribution of specimens subjected to a screening process by non-destructive inspection. We apply the analysis to experimental results on hot pressed Si_3N_4 [6].

2. Crack-size distribution

2.1. Uniaxial tension

The crack-size distribution function of a body subjected to uniaxial tension is given by [7]

$$H_T(d_c) = \exp\left\{-V\left[\left(\frac{\pi}{2}\right)^{1/2}\frac{K_{Ic}}{\sigma_{01}}\right]^{m_1} d_c^{-m_1/2}\right\} \quad (1)$$

where d_c is the crack size which causes fracture, V is the non-dimensional total volume of the body, K_{Ic} is the fracture toughness and m_1, σ_{01} are the shape and scale parameters, respectively. We see from Equation 1 that the relation between $\ln\ln[1/H_T(d_c)]$ and $\ln d_c$ is linear. We have already suggested that we can estimate Weibull's shape and scale parameters using the above relation [7].

2.2. Three-point bending

The crack-size distribution function in the case of three-point bending is given by [1, 2]

$$H_B(d_c) = \int_0^{d_c}\int_{y_t}\int_{x_t}\left[\left(\frac{\pi}{2}\right)^{1/2}\frac{K_{Ic}}{\sigma_{01}}\right]^{m_1} d_c^{-m_1/2}$$
$$\times\exp\left\{-V_{e0}\left[\left(\frac{\pi}{2}\right)^{1/2}\frac{K_{Ic}}{\sigma_{01}}\frac{Lh}{x(h-y)\,d_c^{1/2}}\right]^{m_1}\right\}\mathrm{d}x\mathrm{d}y\mathrm{d}d_c \quad (2)$$

where $V_{e0} = 2bLh/(m_1 + 1)^2$ is the non-dimensional effective volume; b, L and h are the width, lower span and the half length of the height of a specimen, respectively; x and y are the coordinate variables as shown in Fig. 1. x_t and y_t indicate the total domain of each variable. Since Equation 2 cannot be integrated analytically, we do not know whether the $\ln\ln[1/H_B(d_c)]$ against $\ln d_c$ relation is linear or not without doing numerical calculations.

3. Crack-size and strength distribution after non-destructive inspection

First we formulate a new distribution function of crack size after removing those cracks larger than a definite value (threshold value), d_p, by a screening technique which may lead to an improved distribution function of the fracture strength. As an example, we analyse the three-point bending test data of HP-Si_3N_4 [6].

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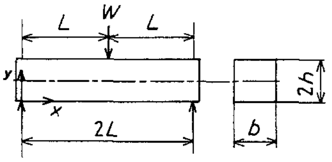

Figure 1 The coordinate systems in the three-point bending test.

3.1. Crack-size distribution function

Suppose that the specimens which contain cracks larger than d_p are removed from a lot by a screening technique, for example, NDI. The crack-size distribution function $H_a(d_c)$ at fracture origin of the remaining specimens for inert circumstances are directly formulated as

$$H_a(d_c) = \frac{\int_0^{d_c} h_B(d_c)\,dd_c}{\int_0^{d_p} h_B(d_c)\,dd_c} \qquad (3)$$

where $h_B(d_c) = dH_B(d_c)/dd_c$.

Fig. 2 shows the calculated results of the $\ln\ln[1/H_B(d_c)] - \ln d_c$ relation, where the Weibull parameters obtained from the three-point bending test of HP-Si_3N_4 [6] are used (see Table I) [8, 9]. Curves I and II in Fig. 2 indicate the theoretical curves before and after screening, respectively, where d_p is taken to be equal to 30 μm. Note that both curves are convex, although that for uniaxial tension (Equation 1) is linear. Crosses and triangles in Fig. 2 are the experimental data plotted by the mean rank method; triangles denote the data obtained by removing values

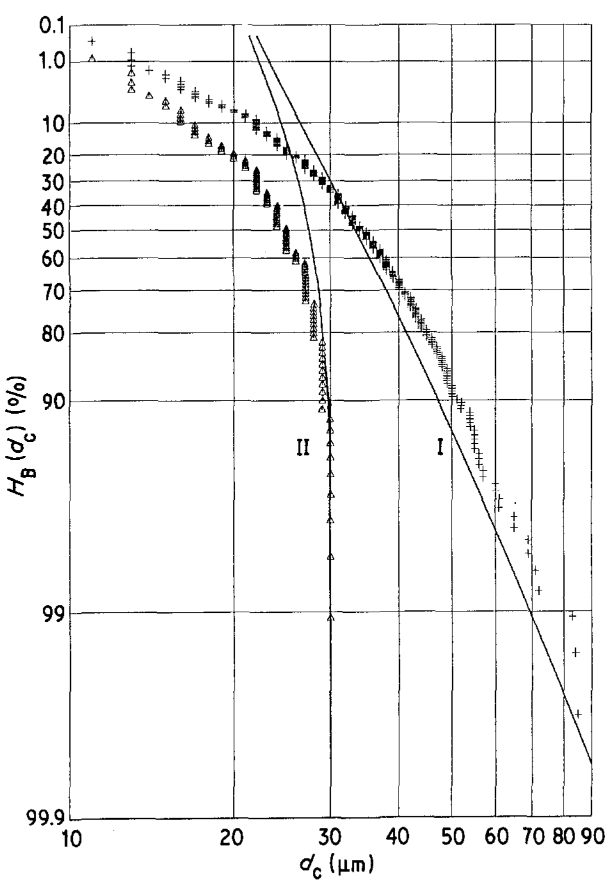

Figure 2 The crack-size distributions (+) before and (△) after NDI. The solid lines are the calculated results. d_p is the threshold crack size in NDI.

TABLE I Weibull parameters and fracture toughness of HP-Si_3N_4

$\hat{m}_1$	$\hat{\sigma}_{01}$ (MPa)	K_{Ic} (MPa m$^{1/2}$)
15.79	959.9	4.06

less than d_p. In the region $d_c > 30\,\mu$m, the theoretical curves almost coincide with the experimental data, which shows the validity of this analysis. In the region $d_c < 30\,\mu$m, the theoretical curve deviates from the experimental data. As stated in the previous report [7], this phenomenon may be caused by the crack-size dependency of the fracture toughness K_{Ic}. Further study is needed on this problem, although the above region is less important for maintaining the structural reliability of ceramic components.

3.2. Strength distribution

For the next step let us analyse the strength distribution after NDI using Equations 2 and 3.

Transforming the variable d_c into the maximum stress at fracture σ_{max}, we obtain a two-parameter Weibull distribution function from Equation 2. On the other hand, from Equation 3 we obtain the following equation (see Appendix):

$$F_a(\sigma_{max}) = \frac{F(\sigma_{max}) - F(\sigma_p)}{1 - F(\sigma_p)} \qquad (4)$$

where

$$F(\sigma) = 1 - \exp\left[-V_{e0}\left(\frac{\sigma}{\sigma_{01}}\right)^{m_1}\right]$$

where σ_p coincides with a proof stress corresponding to d_p for the reason stated in the following.

Suppose that all the specimens which contain cracks larger than d_p are removed from the population. Then the largest crack contained in the remaining specimens is no more than d_p. The smallest strength of the remaining specimens may be realized when the largest crack d_p lies at and perpendicular to the maximally stressed point. Therefore, the smallest strength (σ_p) of the population after NDI (threshold value d_p) is given by the equation

$$\sigma_p = \left(\frac{\pi}{2}\right)^{1/2} \frac{K_{Ic}}{d_p^{1/2}} \qquad (5)$$

σ_p corresponds to the so-called "proof stress" in a proof test. Curve I in Fig. 3 is the theoretical Weibull plot calculated from a two-parameter Weibull distribution function. Curve II in Fig. 3 is calculated from Equation 4 putting $\sigma_p = 930$ MPa, which corresponds to $d_p = 30\,\mu$m. Triangles in the figure are simulated data obtained by omitting those data less than 930 MPa, where the Kaplan–Meier method [10] is adopted. The theoretical Curve II fits well with the simulated data. We see from this figure that σ_p behaves like the proof stress as stated above.

It goes without saying that the above approach is valid for components containing many kinds of fracture origin subjected to an arbitrary stress state.

4. Conclusions

We analysed the crack-size (d_c) and the strength distributions of specimens after NDI by supposing penny-

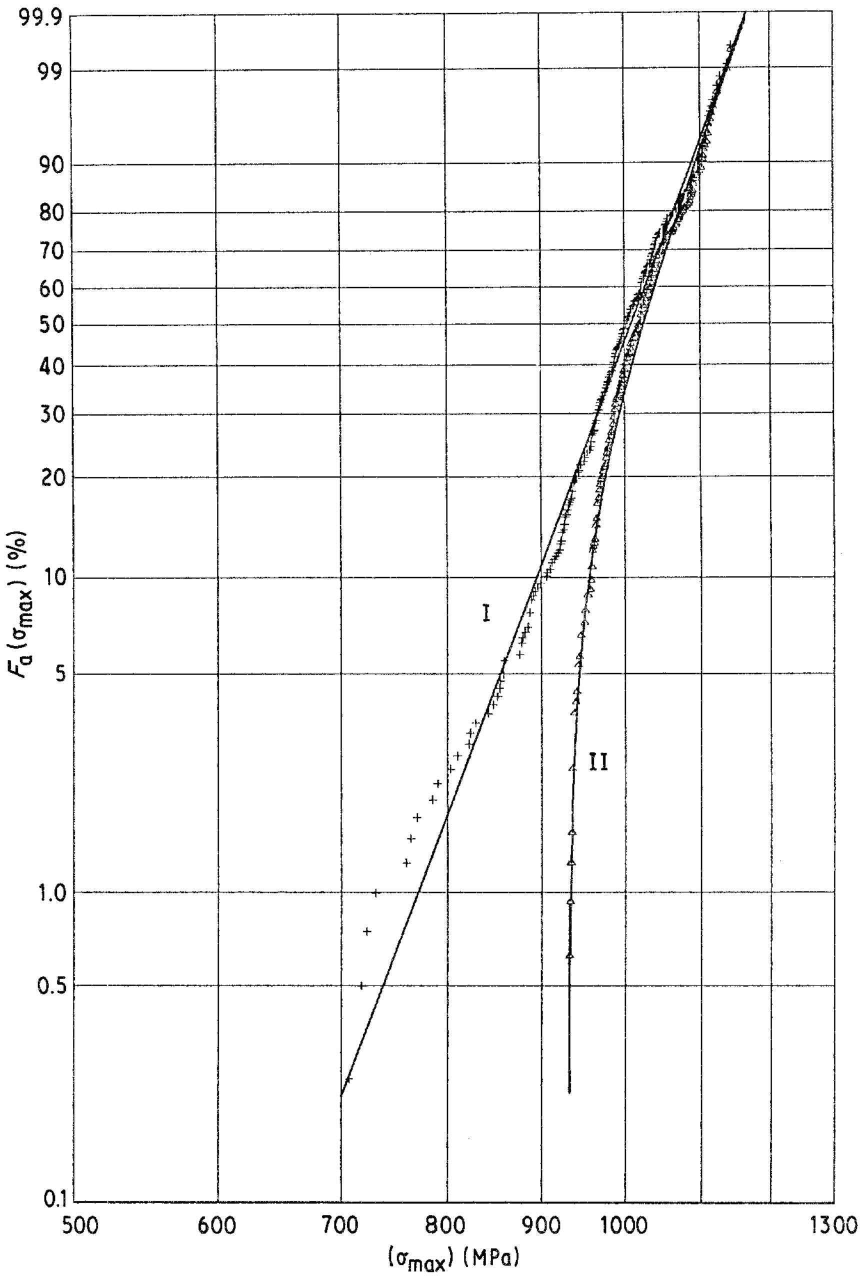

Figure 3 The Weibull plots of the maximum fracture stresses (+) before and (△) after NDI. The solid lines are the calculated results.

shaped cracks to be inner cracks. The results obtained in the work are as follows: the theoretical crack-size distribution function after NDI coincides well with the experimental data for a three-point bending test of HP-Si_3N_4 in the region $d_c > 30\,\mu$m, and the distribution function of the fracture strength calculated from the crack-size data after NDI coincides well with the experimental data. The threshold crack size d_p in NDI corresponds to the proof stress σ_p.

Appendix: Proof of Equation 4

Suppose that penny-shaped cracks are lying perpendicular to the maximum principal stress and that they distribute randomly with respect to their positions and sizes. The joint probability density function for the fracture location (x, y) and the crack size d_c at the fracture origin in the case of the three-point bending of a rectangular cross-sectioned beam specimen is given by the equation [1, 2]

$$h_B(d_c, x, y) = bm_1\left[\left(\frac{\pi}{2}\right)^{1/2}\frac{K_{Ic}}{\sigma_{01}}\right]^{m_1} d_c^{-(m_1+2)/2}$$

$$\times \exp\left\{-V_{e0}\left[\left(\frac{\pi}{2}\right)^{1/2}\frac{K_{Ic}}{\sigma_{01}}\frac{Lh}{x(h-y)d_c^{1/2}}\right]^{m_1}\right\} \quad \text{(A1)}$$

Let us transform the variable d_c into the maximum fracture stress σ_{max} using the relation

$$K_{Ic} = \frac{2}{\pi}\sigma_c\left(\frac{\pi d_c}{2}\right)^{1/2} \quad \text{(A2)}$$

where

$$\sigma_c = \frac{x(h-y)}{Lh}\sigma_{max}$$

and σ_c is the fracture stress at the fracture origin. The Jacobian of the transformation from the variable d_c into σ_{max} is written by

$$J = \frac{\partial d_c}{\partial \sigma_{max}} = \frac{2}{K_{Ic}}\left(\frac{2}{\pi}\right)^{1/2}\frac{x(h-y)}{Lh}d^{3/2} \quad \text{(A3)}$$

Then we obtain the joint probability density function for the fracture location (x, y) and the fracture strength σ_{max} as

$$h_B(\sigma_{max}, x, y) = 2bm_1 \left(\frac{1}{\sigma_{01}}\right)^{m_1} \sigma_{max}^{m_1 - 1} \left[\frac{x(h-y)}{Lh}\right]^{m_1} \times \exp\left[-V_{e0}\left(\frac{\sigma_{max}}{\sigma_{01}}\right)^{m_1}\right] \quad (A4)$$

The marginal with respect to the fracture strength σ_{max} is derived from Equation A4 as

$$\int_0^{\sigma_{max}} \int_0^h \int_0^L h_B(\sigma_{max}, x, y)\,dx\,dy\,d\sigma_{max} = 1 - \exp\left[-V_{e0}\left(\frac{\sigma_{max}}{\sigma_{01}}\right)^{m_1}\right] \quad (A5)$$

which coincides with so-called two-parameter Weibull distribution function.

Since the domain of the variable d_c of the denominator in Equation 3, $(0, d_p)$, corresponds to (σ_p, ∞) for the fracture strength, we obtain

$$\int_{\sigma_p}^{\infty} \int_0^h \int_0^L h_B(\sigma_{max}, x, y)\,dx\,dy\,d\sigma_{max} = \exp\left[-V_{e0}\left(\frac{\sigma_p}{\sigma_{01}}\right)^{m_1}\right] = 1 - F(\sigma_p) \quad (A6)$$

For the numerator in Equation 3,

$$\int_{\sigma_p}^{\sigma_{max}} \int_0^h \int_0^L h_B(\sigma_{max}, x, y)\,dx\,dy\,d\sigma_{max} = \exp\left[-V_{e0}\left(\frac{\sigma_p}{\sigma_{01}}\right)^{m_1}\right] - \exp\left[-V_{e0}\left(\frac{\sigma_{max}}{\sigma_{01}}\right)^{m_1}\right] = F(\sigma_{max}) - F(\sigma_p) \quad (A7)$$

Thus we obtain Equation 4.

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